Cambridge Assessment International Education

# Cambridge International AS & A Level

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*6007163164*

**FURTHER MATHEMATICS** **9231/33**

Paper 3 Further Mechanics **May/June 2021**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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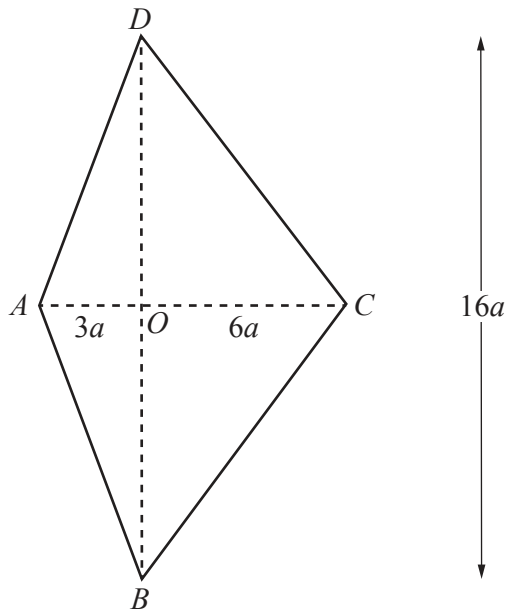
**1**




A uniform lamina $ABCD$ consists of two isosceles triangles $ABD$ and $BCD$. The diagonals of $ABCD$ meet at the point $O$. The length of $AO$ is $3a$, the length of $OC$ is $6a$ and the length of $BD$ is $16a$ (see diagram).

Find the distance of the centre of mass of the lamina from $DB$. [3]

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**2** One end of a light elastic string of natural length 0.8 m and modulus of elasticity 36 N is attached to a fixed point $O$ on a smooth plane. The plane is inclined at an angle $\alpha$ to the horizontal, where $\sin\alpha = \frac{3}{5}$. A particle $P$ of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of $O$. The particle is projected with speed $\sqrt{2}\,\mathrm{m\,s^{-1}}$ directly down the plane from the position where $OP$ is equal to the natural length of the string.

Find the maximum extension of the string during the subsequent motion. [5]



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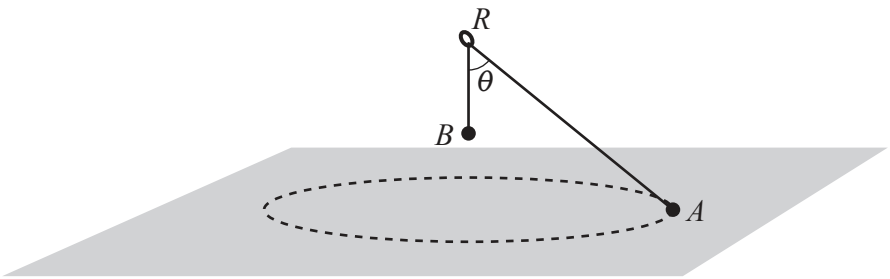
**3**




Particles $A$ and $B$, of masses $3m$ and $m$ respectively, are connected by a light inextensible string of length $a$ that passes through a fixed smooth ring $R$. Particle $B$ hangs in equilibrium vertically below the ring. Particle $A$ moves in horizontal circles on a smooth horizontal surface with speed $\frac{2}{5}\sqrt{ga}$. The angle between $AR$ and $BR$ is $\theta$ (see diagram). The normal reaction between $A$ and the surface is $\frac{12}{5}mg$.

**(a)** Find $\cos\theta$. [3]

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**(b)** Find, in terms of $a$, the distance of $B$ below the ring. [3]



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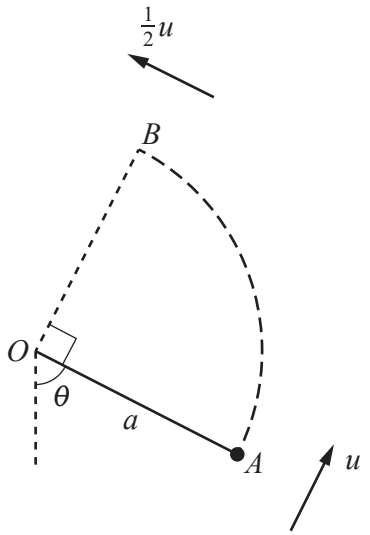
**4**




A particle of mass $m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The particle is initially held with the string taut at the point $A$, where $OA$ makes an angle $\theta$ with the downward vertical through $O$. The particle is then projected with speed $u$ perpendicular to $OA$ and begins to move upwards in part of a vertical circle. The string goes slack when the particle is at the point $B$ where angle $AOB$ is a right angle. The speed of the particle when it is at $B$ is $\frac{1}{2}u$ (see diagram).

Find the tension in the string at $A$, giving your answer in terms of $m$ and $g$. [8]

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**5** A particle $P$ of mass $m$ kg is projected vertically upwards from a point $O$, with speed $20\,\mathrm{m\,s^{-1}}$, and moves under gravity. There is a resistive force of magnitude $2mv$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of $P$ at time $t$ s after projection.

**(a)** Find an expression for $v$ in terms of $t$, while $P$ is moving upwards. [6]



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The displacement of $P$ from $O$ is $x$ m at time $t$ s.

**(b)** Find an expression for $x$ in terms of $t$, while $P$ is moving upwards. [2]

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**(c)** Find, correct to 3 significant figures, the greatest height above $O$ reached by $P$. [2]

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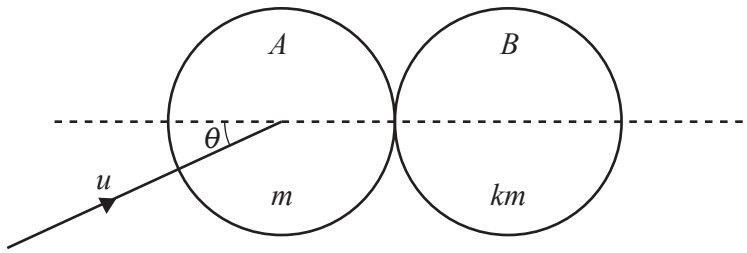
**6**




Two uniform smooth spheres $A$ and $B$ of equal radii have masses $m$ and $km$ respectively. Sphere $A$ is moving with speed $u$ on a smooth horizontal surface when it collides with sphere $B$ which is at rest. Immediately before the collision, $A$'s direction of motion makes an angle $\theta$ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

**(a)** Show that the speed of $B$ after the collision is $\dfrac{4u\cos\theta}{3(1+k)}$. [3]

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70% of the total kinetic energy of the spheres is lost as a result of the collision.

**(b)** Given that $\tan\theta = \frac{1}{3}$, find the value of $k$. [6]



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**7** A particle $P$ is projected with speed $u$ at an angle $\theta$ above the horizontal from a point $O$ on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of $P$ from $O$ at a subsequent time $t$ are denoted by $x$ and $y$ respectively.

**(a)** Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range $R$ in terms of $u$, $\theta$ and $g$. [2]

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**(b)** Deduce an expression for the maximum height $H$, in terms of $u$, $\theta$ and $g$. [2]

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It is given that $R = \frac{4H}{\sqrt{3}}$.

**(c)** Show that $\theta = 60°$. [1]

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It is given also that $u = \sqrt{40}\,\mathrm{m\,s^{-1}}$.

**(d)** Find, by differentiating the equation of the trajectory or otherwise, the set of values of $x$ for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

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## Additional Page

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